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Supporting Mathematics Talk in Kindergarten

Kindergartners are capable of engaging in reasoning about mathematics and justifying their thinking using several resources.

Hala Ghousseini, Sarah Lord, and Aimee Cardon

Ms. Sanders's (all names are pseudonyms) kindergarten students are gathered on the floor for their morning calendar activity. They have just counted the total number of days they have been in school. It has been 129 days. The class is particularly focusing on the number 29 and different ways to represent it. Students first represent it with bundles of sticks and with base-ten blocks. Then they turn to representing it with tally marks: five groups of five tally marks and four individual tally marks. A student, Jenna, is counting the total number of tally marks, "5, 10, 15, 20, 25, 26, 27, 28, 29." Ms. Sanders interjects and addresses the class, "So, I notice [Jenna] is counting these last four by ones. So, when we get over here [*pointing to 25*], why is Jenna counting by ones instead of counting by fives?"

As several students raise their hands, Ms. Sanders calls on Ryan, who explains, "Because they are, uhm, ones, and you don't count by fives." Then, pointing to the representation with bundles of sticks, which had been completed earlier, he continues, "like these!"

Ms. Sanders inquires, "So you are thinking about earlier when we could not count our sticks by tens anymore, and we had to count by ones?"

Ryan nods in agreement and reaches for some base-ten blocks (see figure 1), selecting a tens block and

a ones block. He says, "This [ones block] is smaller than this one [tens block]." Then pointing to the representation with the tally marks, he signals first to a group of five tally marks and says, "Five," and then to the last four ungrouped tally marks and says, "Four." The teacher asks the class, pointing to the four tally marks, "Do we have a group of five here?" and the students say in chorus, "No."

Fig. 1



The teacher can support young students' efforts to communicate mathematical ideas with academic language and by supplying number lines, pictures, number charts, and other representations in the classroom.

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This example reflects how kindergartners are capable of engaging in reasoning about mathematics and justifying their thinking using several resources. Ryan was justifying the claim that after reaching 25, the count continues by ones because there are no more groups of fives. As a young learner still developing his language skills, Ryan may not yet have the proficiency to express his thinking using academic terms. In addition to formal and informal linguistic resources, he relies on gestures and representations around the classroom, which he uses to argue for the difference between a group of five tally marks and four individual tally marks.

Classroom discourse is integral to mathematics instruction at all levels. The expectation that all students will engage in mathematics discourse is central to the Common Core State Standards, which emphasize practices like conjecturing, justifying, and reconciling different ideas to analyze a problem situation (NGA Center and CCSSO 2010). This work is challenging for learners at any age, but perhaps especially for very young learners who are still developing their general oral language skills and at the same time beginning to acquire academic language. Consequently, the mathematical development of young learners is intertwined with the development of their language and communication skills (Greenes, Ginsburg, and Balfanz 2004).

Research suggests that children as early as kindergarten can consider alternative strategies and are capable of sophisticated mathematical thinking (Carpenter et al. 2014). To engage in this type of mathematical activity, students need to be able to communicate about their mathematical reasoning in ways that others can understand and respond to. This involves their use of both oral language and gestures. In addition, young students' efforts to communicate mathematical ideas can be supported in powerful ways by available representations in their classroom (e.g., number lines, pictures, and number charts). The classroom teacher plays a vital role in providing opportunities for students to communicate their mathematical reasoning with academic language and supporting their efforts to represent their emerging ideas using available classroom resources.

We draw on data from a larger study of teachers' facilitation of classroom discourse in elementary classrooms to highlight this vital role of the teacher in supporting students' take up of academic language and encouraging them to use classroom resources in support of their communication. We elaborate three important practices that a kindergarten teacher used to support students' mathematics discourse. We focus on the

work of a kindergarten teacher, Ms. Sanders, because we saw in her classroom consistent evidence of kindergartners participating in socially and intellectually demanding whole-class mathematics discussions. We observed and videotaped five of Ms. Sanders's mathematics lessons over a period of four months and interviewed her after each lesson. As we analyzed videos and transcripts of class discussions and interviews, we attended to the practices Ms. Sanders used to facilitate mathematical discourse and develop intellectual community among her students. Our analysis yielded three important practices that we highlight in this article with related sentence frames for teachers (see figure 2): (1) establishing expectations that support mathematical discourse, (2) eliciting student thinking, and (3) narrating student thinking. In what follows, we use vignettes from Ms. Sanders's kindergarten classroom to elaborate the nature of these practices and to illustrate how the practices supported the efforts of young learners to communicate about mathematical ideas in powerful ways.

ESTABLISHING EXPECTATIONS FOR MATHEMATICAL TALK

Ms. Sanders set expectations in two different ways, which directly contributed to the success of the mathematical discussions in her classroom. The first expectation established the norm of explaining one's thinking. Using phrases like, "Be ready to share with us how you knew that your answer was correct," or "We are going to look closely at Katie's work and talk together about her thinking," Ms. Sanders often reminded her students well ahead of a discussion that they should prepare to share their ideas. These reminders occurred throughout the lesson, including when she was giving directions for an independent task and when she was consulting with students working independently or with partners.

Another expectation that Ms. Sanders promoted was the use of available classroom resources to support mathematical explanations, including charts and number lines, pictorial representations, and manipulatives. Students knew that they could draw on these resources as they endeavored to communicate their thinking. In our analysis, we noted three specific ways Ms. Sanders supported her students' mathematics discourse with resources:

- 1. She made resources available around the classroom and physically within the students' reach.
- 2. She encouraged students to move freely around the room to seek resources to support their thinking.

3. She frequently oriented students to resources they could use in supporting their explanations.

In our data, we saw examples of students' use of resources both as a direct response to the teacher's suggestion and as a result of their own spontaneous initiative. As the example in the introduction to this article illustrates, when Ms. Sanders asked Ryan why Jenna was counting by ones instead of counting by fives when she reached 25, he independently reached for bundles of sticks and base-ten blocks to explain his thinking. He used these classroom resources to argue that when one gets to 25, enough ones are not present to make a group of five. He stated, "Because they are, uhm, ones, and you don't count by fives." His actions reflected the routine ways the students interacted with resources in Ms. Sanders's classroom, which we will further illustrate in the next section.

ELICITING STUDENT THINKING

In Ms. Sanders's classroom, students' participation in mathematics discourse was structured around several mathematical tasks that engaged them in noticing shapes and patterns and making sense of magnitudes and relationships among numbers. After posing a task, Ms. Sanders consistently elicited student thinking using two forms of questioning. Her elicitation regularly started with open-ended questions that solicited students' initial ideas and explanations and surfaced the representations they drew on, such as "How do you know? What do you notice?" She then followed with more probing questions that targeted mathematical ideas (e.g., Why did you start with 9?), language precision (e.g., What do you mean by "drew them the same way"?), and representations (e.g., What do those dots mean here?). Our analysis revealed that starting with open-ended questions allowed the kindergartners to choose several representations as contexts to support their explanations. The probing questions, in turn, pressed students to further describe and articulate the mathematical ideas they were attempting to convey using several representations. This sequencing of open-ended and more probing questions is an approach that researchers have also shown to support the development of mathematics discourse among emergent bilinguals (Banse et al. 2016).

To illustrate this process, we return to the lesson we featured in the introduction, in which the class was figuring out the number of days they had been in school. It had been 129 school days. We rewind to the part where the class was representing this number on a place-value

Fig. 2

Establishing expectations for mathematical discourse

- Remember we will be sharing our ideas when we finish working.
- We are going to listen to [student]'s thinking and then talk about their ideas.
- You can use [these resources] to explain your idea if it helps.
- [These resources] are available to help you while you work and to help explain your thinking.

Eliciting student thinking

- Can someone else add on to what [student] said?
- What does it mean when [student] says [math idea]?
- How do you know [math idea]?
- How does [student] know [math idea]?
- What do you notice about [math idea]?

Narrating student thinking

- [student] told us that [math idea]
- [student] noticed [math idea]
- I noticed that [student] said [this...]
- [Student] is showing us [this...]

These three practices and possible sentence stems can support mathematics talk.

pocket chart with bundled sticks, grouped in hundreds and tens. The class had just collectively counted nine sticks in the ones pocket (listen to audio 1). Ms. Sanders asked, "How many days until we make another group of tens?"

Audio 1: Listen to Students Explaining How Many Days until They Make Another Group of Tens.

In this example, the open-ended nature of Ms. Sanders's first question provided an opportunity for Aiden to identify and draw on resources that he appeared to view as relevant to his sense making: He reached out to the hundred chart and used it to elaborate his reasoning, using language that suggested that he was treating the chart as a number line where "one more day" was represented as a "hop" from 9 to make a group of 10. The series of probing questions that Ms. Sanders posed in response to Aiden's initial statement supported him in clarifying his reasoning and making visible the ways in which he was mentally acting on numerical objects. In this process, Aiden began by treating 9 and 10 as two points on the number line. As he responded to Ms. Sanders's questions, his language became more precise, referring explicitly to the place value of the digits in 9 and 10. After establishing that the number 9 meant 9 ones, Aiden's reference to the hop suggested that he was adding an additional 1 to make a group of 10-using the expression "a group of 10" in his response.

Additionally, this exchange demonstrates how students responded to Ms. Sanders's consecutive prompts by appealing to several different representations to which they had access. Aiden and Ryan relied on both the pocket chart with bundles of tens and ones and the hundred chart to share and clarify their ideas about place value. The mathematical activity demonstrated by Aiden and Ryan in this exchange-well-grounded in representations-is important for young children's mathematical development as they learn to make decisions about how to express their reasoning (Russell et al. 2017). It also benefits children's language development as students use representations as language proxies and take up the teacher's more academic language as they gain experience communicating their mathematical ideas.

Ms. Sanders's elicitation and pressing for student thinking in the context of Aiden's strategy also engaged more than one student. Her invitation to students to apply their thinking to Aiden's reasoning positioned the work of responding to his thinking—interpreting it and making it visible—as a collective responsibility of the class. In the process, Ms. Sanders attended to students' understanding of important mathematical ideas and to shared language and representations that can support this work. She oriented students to Aiden's idea by asking them *how he knew* that he has a group of tens, a focal, and complex, mathematical idea for students' mathematical development at this stage (Carpenter et al. 2014). She also infused their exchanges with precise mathematical language, "Can someone add on, using our words, 'the tens place' and 'the ones place'?"

NARRATING STUDENT THINKING

Ms. Sanders also fostered a rich discourse community within her kindergarten classroom by narrating student thinking. Narration involves providing a running commentary that describes what a student says and does (or may have said or done prior to speaking). Young learners draw on their knowledge of oral language (Strickland 2006) and multimodal forms of communication to support their attempts to convey coherent accounts of their thinking (Ball 1993; Dunphy 2015). In this context, narration is a particularly important type of support that teachers can offer. Beyond revoicing, narration serves to integrate a student's verbal and nonverbal attempts at communication into a holistic narrative that promotes their conceptual understanding.

Ms. Sanders's practice of narration is evident in the preceding example. When Ms. Sanders asked Aiden to clarify what he meant by "this is where we start," Aiden mainly pointed to the place-value pocket chart and said, "That's our one." At this moment, Ms. Sanders engaged in an act of narration in which she not only repeated Aiden's words but also illustrated the connections he was making between the hundred chart and the place-value chart. She started by pointing to the place-value chart, "So, Aiden is looking at the ones place, and he found that we have nine ones today." Then she pointed to the hundred chart to connect what Aiden said to his initial claim, "So, he's starting at nine, and then what did you say next, Aiden?" Simultaneously, through her gestures to the different representations that the Aiden used, Ms. Sanders integrated into her narration his nonverbal embodied sense making (Alibali and DiRusso 1999) and provided language support through her connection to representations as visual cues (Cady, Hodges, and Brown 2010).

Additional instances were observed in which Ms. Sanders used narration to highlight mathematical processes evident in student work. In another lesson, for example, Ms. Sanders was helping students build an understanding of the term *equals* to mean "the same amount." As part of this lesson, students completed a task in which they were to first give an equal number of snowballs to both a penguin and a polar bear and then represent the equality with an equals sign.

After students completed their work on this task, Ms. Sanders regathered the students for a whole-group discussion in which several students shared their work using a document camera. One of these students, Kara, shared how she gave eight snowballs to the bear and eight to the penguin. Evident on her notebook were semivisible dots representing snowballs she had erased (see figure 3). Ms. Sanders used narration to highlight the nature of these semivisible dots. She started by noting that they represented snowballs that Kara had erased. Then she continued.

Ms. Sanders: OK, so we see the 8 there. Now, I want to show you something that Kara did. I was thinking, "Oh, my goodness, Kara is really thinking hard about her math today!" At first, when Kara was working, the polar bear had 8 [she points to the polar bear's snowballs] and the penguin had 10 [pointing to the penguin's snowballs]. And, we were talking, and we said, "Hmm, well, if the polar bear has 8 and the penguin has 10, do they have the same amount?"



Kara's snowballs representation showed snowballs she had erased.

Students: No! [in chorus]

Teacher: And [Kara] said, "Oh! I need to fix that. I need to take some away." And that's why we see, when she's thinking hard about her math, and she said, "Oops!" and took those away [*pointing to the semi-erased dots*], and she double checked: They both have 8.

The narration support that is evident in this excerpt suggests the way Ms. Sanders used it to position students' strategies and the processes evident in their work as resources for the classroom community. By highlighting the way Kara revised her work, Ms. Sanders was normalizing the process of revising as an aspect of doing mathematics (Lampert 2001). In her narration, she framed this work as an aspect of "thinking hard" about the mathematics in relation to the meaning of "Do they have the same amount?" Ms. Sanders assigned agency to Kara in the process, centering Kara as the learner who made particular choices as she worked on this task.

CONCLUSION

We know from prior research in early childhood contexts that children have a natural inclination to engage in a wide range of mathematical activities such as counting, patterning, and developing spatial relationships. Our study shows that when teachers offer appropriate opportunity and support, children of this age can also engage in rich classroom discourse that involves communication about emerging mathematical concepts and ways of reasoning. Whether young children are involved in play-based learning or are learning in a more "academic" setting, we believe that teachers' practices can give essential support for children's sense making and understanding (Pyle and Danniels 2017). By using the teaching practices we highlight here, teachers can foster the development of young learners-as mathematical thinkers, as communicators, and as members of an intellectual community-in a holistic and integrated way. 🗕

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